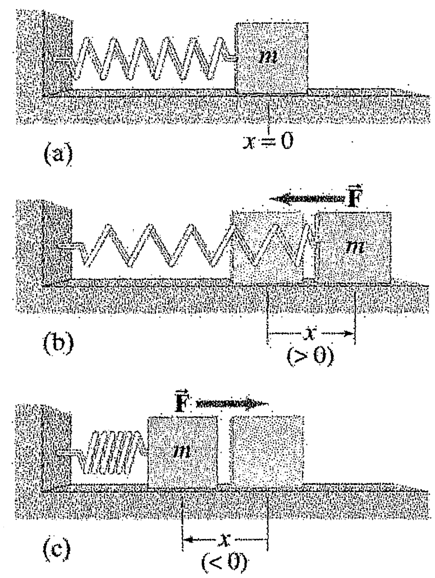
KEY

Mass-spring system Problem

A block of mass ***m=5 kg*** is attached to a linear spring of constant ***k =140N/m***. The mass is placed on a frictionless surface. An external force is applied pulling the mass to a distance +***A = 10 cm*** on the table. Refer to the diagram and answer the following:



(a) Calculate the average force applied displacing the mass to 10 cm

$F_{avg} = \frac{F_{max}}{2} = \frac{kA}{2} = \frac{140 \times 0.1}{2} = 7N$

(b) What is the work done on the mass-spring system by the external agent (force)?

$W = F_{avg} \cdot A = 7 \times 0.1$

$= 0.7 J$

(c) What is the Potential energy of the system when the block is at:

(i) +A and – A

$U_s = \frac{1}{2} k \Delta x^2 = \frac{1}{2}(140)(0.1)^2$

$U_s = 0.7 J$

(ii) Equilibrium

$U_s = \emptyset$ since $\Delta x = \emptyset$

(iii) Δx = 5 cm

$U_s = \frac{1}{2} k \Delta x^2 = \frac{1}{2}(140)(0.05)^2$

$U_s = 0.175 J$

(iv) Δx = - 5 cm

$U_s = 0.175 J$

(d) What is the kinetic energy of the system when the block is at:

(v) +A and – A

$K = \emptyset$ since $v = \emptyset$ at amplitude.

(vi) Equilibrium

$E_i = E_f$

$\frac{1}{2} k A^2 = \frac{1}{2} m v^2 = 0.7 J$

(vii) Δx = 5 cm

$E_i = E_f$

$\frac{1}{2} m v^2 = \frac{1}{2} k A^2 - \frac{1}{2} k \Delta x^2 = 0.525 J$

(viii) Δx = -5 cm

$\frac{1}{2} m v^2 = 0.525 J$

(e) What is the instantaneous acceleration of the block when it is just released from +A?

$$F = ma = kA$$
$$a = \frac{kA}{m} = \frac{(140)(0.1)}{5} = 2.8\ m/s^2$$

(f) What is the instantaneous acceleration of the block when it is at 5 cm from equilibrium?

$$F = ma = k\Delta x$$
$$a = \frac{k\Delta x}{m} = \frac{(140)(0.05)}{5} = 1.4\ m/s^2$$

(g) What is the acceleration of the block when it is at equilibrium? Justify your answer.

$$a = 0 \quad \text{since} \quad \frac{k\Delta x}{m} \quad \Delta x = 0$$

(h) What is the velocity of the block when it is at equilibrium?

$$\frac{1}{2}mv^2 = \frac{1}{2}kA^2$$
$$v = \sqrt{\frac{kA^2}{m}} = 0.53\ m/s$$

(i) What is the velocity of the block when it is 5 cm from equilibrium?

$$K = 0.525\,J = \frac{1}{2}mv^2$$
$$v = \sqrt{\frac{kA^2}{m} - \frac{k\Delta x^2}{m}} = 0.46\ m/s$$

(j) At what location Δx, the kinetic energy of the block exactly equals to the elastic potential energy.

$$E_i = E_f \qquad K = U \Rightarrow U = \frac{1}{2}k$$
$$\frac{1}{2}kA^2 = K + U$$
$$\frac{1}{2}kA^2 = \frac{1}{2}k\Delta x^2 + \frac{1}{2}k\Delta x^2 \Rightarrow A^2 = 2\Delta x^2$$
$$\Delta x = \sqrt{\frac{A^2}{2}} = \frac{A}{\sqrt{2}} = 0.07\ m$$

(k) What is the velocity of the block when the kinetic energy exactly equals to the elastic potential energy of the block.

$$\frac{1}{2}mv^2 = \frac{1}{2}kA^2 - \frac{1}{2}k\Delta x^2$$
$$v = \sqrt{\frac{kA^2 - k\Delta x^2}{m}} = 0.37\ m/s$$