

Mass-spring system Problem

KEY

A block of mass m is attached to a linear spring of constant k . The mass is placed on a frictionless surface. An external force is applied pulling the mass to a distance $+A$ on the table. Refer to the diagram and answer the following:

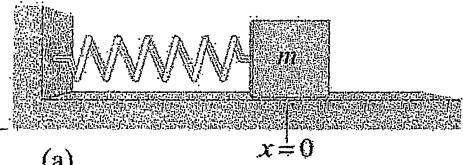
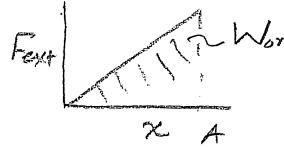
- (a) Derive an expression for the total mechanical energy of the system.

Mechanical energy will come from the work done

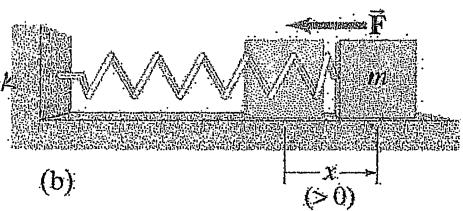
$$W = \frac{F_{ext} \cdot A}{2} \quad F_{ext} = -F_s = kA$$

$$W = \left(\frac{KA}{2}\right)A = \frac{1}{2}KA^2$$

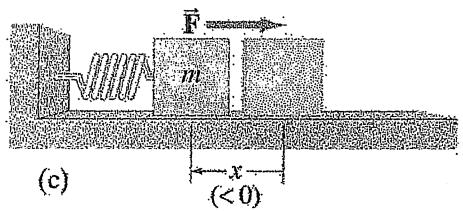
$$U_s = \frac{1}{2}KA^2$$



(a)



(b)



(c)

- (b) What is the potential energy of the system when the block is at:

- (i) $+A$ and $-A$

$$U_A = U_{-A} = \frac{1}{2}KA^2; \text{ since } K = 0 \text{ at } \pm A$$

- (ii) Equilibrium

$$U_s = 0; \text{ since } \Delta x = 0$$

- (iii) $\Delta x > 0$

$$E_i = E_f \quad u = 0$$

$$U_{si} = U_{sf} + K_f \quad U_{sf} = \frac{1}{2}KA^2 - \frac{1}{2}mv^2 \Rightarrow U_{sf} = \frac{1}{2}KA^2 - \frac{1}{2}mv^2$$

- (iv) $\Delta x < 0$

$$E_i = E_f \quad u = 0$$

$$U_{si} = U_{sf} + K_f \quad U_{sf} = \frac{1}{2}KA^2 - \frac{1}{2}mv^2 \Rightarrow U_{sf} = \frac{1}{2}KA^2 - \frac{1}{2}mv^2$$

- (c) What is the kinetic energy of the system when the block is at:

- (v) $+A$ and $-A$

$$K = 0 \text{ since } v = 0 \text{ at } \pm A$$

- (vi) Equilibrium

$$K = \frac{1}{2}KA^2; \text{ since } \Delta x = 0 \text{ at equilibrium}$$

- (vii) $\Delta x > 0$

$$E_i = E_f \quad u = 0$$

$$U_{si} = U_{sf} + K_f \quad K_f = \frac{1}{2}KA^2 - \frac{1}{2}K\Delta x^2 \Rightarrow K_f = \frac{1}{2}K(A^2 - \Delta x^2)$$

- (viii) $\Delta x < 0$

$$E_i = E_f \quad u = 0$$

$$U_{si} = U_{sf} + K_f \quad K_f = \frac{1}{2}KA^2 - \frac{1}{2}K\Delta x^2 \Rightarrow K_f = \frac{1}{2}K(A^2 - \Delta x^2)$$

- (d) Derive an expression for the instantaneous acceleration of the block when it is just released from +A.

$$F_s = -kA$$

$$F_s = ma$$

$$ma = -kA$$

$$a = \frac{-kA}{m}$$

- (e) What is the acceleration of the block when it is at equilibrium? Justify your answer.

The acceleration will be zero at equilibrium since there will be no spring force F_s at equilibrium

$$F_s = 0 \quad \text{since } \Delta x = 0$$

$$a = 0$$

- (f) At what location Δx , the kinetic energy of the block exactly equals to the elastic potential energy.

$$E_k = E_p \quad v = 0$$

$$\frac{1}{2}KA^2 = K + U$$

$$K = U$$

$$\frac{1}{2}KA^2 = 2U$$

$$\frac{1}{2}KA^2 = 2\left(\frac{1}{2}K\Delta x^2\right)$$

$$\frac{1}{2}KA^2 = K\Delta x^2 \Rightarrow \Delta x = \frac{A}{\sqrt{2}}$$

- (g) What is the speed of the block when the kinetic energy exactly equals to the elastic potential energy of the block.

$$E_k = E_p \quad v = 0$$

$$\frac{1}{2}KA^2 = K + U$$

$$\frac{1}{2}K = U$$

$$\frac{1}{2}KA^2 = 2K$$

$$\frac{1}{2}KA^2 = 2\left(\frac{1}{2}mv^2\right)$$

$$\frac{1}{2}KA^2 = mv^2 \Rightarrow v = A\sqrt{\frac{K}{2m}}$$